Developing Mathematical Thinking and Problem Solving Ability

Overview
- NCTM Problem Solving Standard
- Polya’s Problem Solving Process
- Components of a Problem Solving Instructional Program
- Assessing Problem Solving
- Problem Solving Scenarios

NCTM Problem Solving Standard
Instructional programs from prekindergarten through grade 12 should enable all students to-
- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Problem Solving
- Are you ready to tackle a math problem with confidence?
- Do you have a briefcase filled with problem-solving strategies that help when you encounter a new problem?
- Do you get confused about how to solve problems?

Problem Solving Steps by George Polya
- UNDERSTAND the problem
- DEVISE A PLAN for solving it
- CARRY OUT solve the problem
- LOOK BACK and check your solution

UNDERSTAND
- Restate the problem
- Highlight or identify important facts
- Determine the question or problem to be solved

DEVISE A PLAN
- making a table, chart or graph
- using estimation
- making a list
- acting out the problem
- guess and check
- working backward
- finding a pattern
- solving multi-step problems
- drawing pictures or diagrams
- interpreting remainders
- making and using model
- using formulas
- choosing an operation
- solving a simpler problem
- writing a number sentence or equation
SOLVE

• Identify Needed Facts
• Choose an Appropriate Strategy
• Pencil and Paper
• Calculator
• Addition, Subtraction, Multiplication, Division
• Show ALL Work

LOOK BACK

• Restate the Question
• Check the Answer
• Does the Answer Make Sense?
• Record the Answer
• Add Necessary Units or Labels

Before Actions

• Make sure students understand the problem.
• Read the problem to the class or have a student read it aloud.
• Make sure students understand the problem and what is being asked.
• Discuss terms that might be unclear.
• Brainstorm possible solution strategies.
• Clarify the task.

During Actions

• Circulate around the room observing and questioning students about the strategies they are using.
• Ask students questions to help them clarify the direction their solution process is taking them.
• Provide hints to students who are stuck.
• Make sure students are answering the question asked in the problem.

After Actions

• Encourage students to reflect on their solution and the problem solving process they used.
• Make sure to emphasize the process as well as the answer.
• Encourage all students to participate in the discussion.
• Encourage all students to communicate through words, diagrams, pictures and/or manipulative materials.

Guidelines for Teaching Problem Solving

• Provide hints and directions rather than solutions.
• Encourage – risk taking – the use of models by students – verbal expression – perseverance
Guidelines for Teaching Problem Solving

- Listen and accept ideas - provide guidance not censorship
- Provide successful problem solving experiences.
- Help students to:
  - evaluate their own solutions
  - develop their own heuristic behaviors
  - develop the belief that “they can do it.”

Classroom Environment for Problem Solving

- Build in success
- Praise effort and risk taking
- Use non-evaluative responses
- Listen to many students
- Use cooperative work
- Show enthusiasm toward problem solving
- Model problem solving behaviors

Assessing Problem Solving

- Presenting students with a problem-solving situation and observing how they meet it.
  - Observations
- Interviewing students
  - Interviews
- Having students describe to a group how they solved the problem
- Having one student teach another how to solve a problem
- Inventories and Checklists
- Paper and Pencil Tests

Resources

- One Riddle, One Answer by Lauren Thompson - http://lswstudio.homestead.com/books.html
- TMC Problem Solvers: http://my.nctm.org/resources/jcs.org?journal_id=4&issue_id=775
- Math is Everywhere: A Problem Solving Teaching Unit - http://www.pide.edu/public/corriculum/units/94007/94007.16-z.html
- Figure This - http://www.figurethis.org/
- Navigating Through Problem Solving and Reasoning in Grade 3 by Karol Yeads http://my.nctm.org/EBusiness/ProductCatalog/product.aspx?id=12719
- Navigating Through Problem Solving and Reasoning in Grade 4 by Karol Yeads http://my.nctm.org/EBusiness/ProductCatalog/product.aspx?ID=12886
- Navigating Through Problem Solving and Reasoning in Grade 5 by Karol Yeads http://my.nctm.org/EBusiness/ProductCatalog/product.aspx?ID=13012

Summary

- Problem solving MUST be an integral part of mathematics program.
- Mathematics programs must include different types of problems.
- Teachers must teach problem solving strategies.

Problem Solving Scenarios

- Work with a partner
- Identify the problem
- Choose a strategy to solve the problem
- Solve the problem
  - Show how you solved the problem
- Look back on the solution
- Be prepared to share your problem, the strategy and the solution.