The significance of problem solving in the K–12 mathematics curriculum has been well documented over the years by professional organizations and individuals. The National Council of Supervisors of Mathematics (NCSM 1977) and the National Council of Teachers of Mathematics (NCTM 1980, 1989, 2000) endorse the inclusion of problem solving at all grade levels. For example, NCTM (1989) states, “The development of each student’s ability to solve problems is essential if he or she is to be a productive citizen” (p. 6). In the same publication, NCTM asserts that “problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematical activity. Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned” (p. 23). NCTM reiterates its position on problem solving in Principles and Standards for School Mathematics (NCTM 2000).

Cognitively Guided Instruction (CGI)

Cognitively Guided Instruction (CGI) is an example of a research-based, problem-solving approach to teaching mathematics. As the name implies, an understanding of the learner’s thinking guides instruction. The idea originated at the University of Wisconsin with Thomas Carpenter and his colleagues. Although it is intended primarily for grades K–3, CGI is such a broad philosophy of teaching that it can be adapted for use at almost any grade level.

In this approach, the teacher presents students with mathematical word problems set in the context of their environment and allows students the freedom to create their own strategies for solving the problems using available resources. They can use physical, pictorial, or symbolic representations and are encouraged to explore different ways to solve a particular problem. In the process of solving problems, children invent their own algorithms and share their problem-solving strategies with the rest of the class. The process of obtaining an answer is more important than the answer itself.

Based on a student’s response to a problem, the teacher prepares appropriate follow-up problems. For example, if a student has not solved a problem satisfactorily, the teacher may give him or her a less difficult problem, perhaps one with lesser numbers. The CGI approach easily addresses NCTM’s Process Standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation); this is one of its greatest benefits. More important, learners become adept at these processes.

The CGI teacher is a facilitator of learning rather than a disseminator of knowledge. He or she avoids imposing adult thinking on students. The teacher spends a lot of time questioning and listening to students’ explanations in order to determine...
how they are thinking. Therefore, the teacher must possess good questioning skills. Once the teacher has determined how the learner is thinking, he or she tries to build on the learner’s ideas. CGI is based on the premise that children come to school with a great deal of mathematical knowledge; the task of the teacher is to uncover this knowledge and build on it.

CGI places great emphasis on individualization, conceptual understanding, and higher-order thinking. More emphasis is placed on formative evaluation than on summative evaluation. The approach does not use any teaching materials such as textbooks, so the teacher must prepare his or her own problems. This and other responsibilities such as classroom management make teaching challenging for the CGI teacher. For a discussion of the characteristics of a CGI classroom, see Chambers and Hankes (1994).

Classroom management issues are important concerns for the CGI teacher, especially if the class is large, because CGI is an individualized approach. If students are properly oriented to CGI, however, they become independent, responsible, and accountable learners and, therefore, can work on their own while the teacher is attending to one student. Independence, responsibility, and accountability take time to develop. From the inception of instruction, the teacher must make students aware of what is required of them and implement policies consistently. Allowing students to make their own decisions facilitates development of these qualities. For example, students choose strategies that are meaningful to them to solve a problem, and they select what materials to use. They do not depend on explicit instructions from the teacher. Instead of correcting students’ mistakes, the teacher can help students correct their own mistakes by questioning, pointing out contradictions, providing counterexamples, suggesting that a result be checked against the conditions in the problem, or asking another student to share his or her perspective of the task. The teacher can teach students basic research skills to answer their own questions; in this way, the teacher is relieved of the responsibility to answer students’ questions. Students become accountable because they know that the teacher may call on anyone to explain his or her solution to
a problem. Students can help one another stay focused by identifying off-task behaviors and dealing with them. Students can develop independence, responsibility, and accountability only if the teacher gives them opportunities to do so.

The presence of a paraprofessional in the classroom helps alleviate management problems. The first author of this article observed many CGI classrooms in North Carolina and did not recognize any serious management problems. The teacher in the investigation described in this article did not have any problems managing the class. From the beginning, she made students aware of her expectations and the students cooperated. She also used the services of the paraprofessional.

In the mid- to late nineties, workshops to train teachers to use CGI were conducted in North Carolina; previously, workshops were offered in Wisconsin. Two national conferences were devoted entirely to CGI. Presentations on CGI often are made at national, regional, and state conferences. We are not aware of a single organization that coordinates CGI workshops and conferences. Readers who are interested in CGI, however, may want to search online for “Cognitively Guided Instruction” to get information about possible CGI workshops and the names of contacts. The first author of this article is fully trained to conduct CGI workshops and was a co-leader of some of the North Carolina workshops.

### CGI Problem Types

Four broad CGI problem types are related to addition and subtraction: join, separate, part-part-whole, and compare. Each of these has sub-types. For a detailed description of the problem types and children’s strategies, see Carpenter et al. (1999), which includes a CD that illustrates children working with these problem types. Knowledge of the problem types helps the teacher present a balanced curriculum to students and move students to higher levels of thinking. Some types of problems are more difficult than others.

“Separate” problems have three sub-types: result unknown, change unknown, and start unknown. The following is an example of a “separate change unknown” problem: “Mary had 9 cookies. After she gave some of these cookies to Jon, she had 5 cookies left. How many cookies did she give to Jon?”

This article reports the results of an investigation into the problem-solving strategies that a group of first graders used when solving “separate change unknown” problems. The children had already done some work with “separate result unknown” problems, and the teacher thought that they were ready to move on to “change unknown” problems. Knowledge of these strategies will help teachers plan more appropriately for students.

### Procedure

The teacher, the second author of this article, read the problem to each child and reread it if necessary. Sometimes, the teacher and the child read the problem together. The teacher asked the child to solve the problem using any method and to write an explanation of the solution. The child could use materials such as counters, number lines, markers, and drawing paper. After the child’s initial solution, the child gave an oral explanation of the solution process during a dialogue between the teacher and the child. The usefulness of dialogue with learners is well documented; see, for example, Hoosain (2000) and Buschman (2001). The data were collected through the teacher’s observations and dialogues with the students. The students’
responses follow; their names have been changed.
The teacher gave John the following problem:

There were twenty orange pumpkins on the
backyard fence. Some of them fell down; now there are twelve. How many pumpkins fell
down?

The teacher read the problem with John and observed how he solved it. He carefully counted
twenty counters; then he took some away and said that the answer was 9. When the teacher asked him
to explain how he obtained that answer, he repeated the procedure with the counters. Then he said, “I started with twenty, then separated twelve because that was what was left, and I got eight.” He
discovered that his mistake was in counting.

The teacher asked John to write an explanation
of how he thought about the problem. He wrote, “20 – 12 = 8.” Recognizing that the equation did
not reflect the information in the problem, the
teacher read the problem again with the student
and asked him to explain how he got his subtrac-
tion sentence. He smiled as he crossed through the
first sentence and replied, “It is wrong.” In his second attempt, he wrote, “20 – 8 = 12.” He explained,
“The word some gets a box because I do not know
what it is.” His explanation could be a reflection of
his prior knowledge of how to deal with key words
in a problem. Perhaps the teacher, who is tran-
sitioning from a traditional approach to a problem-
solving approach, taught him that he must write the
semantic equation, 20 – 8 = 12, instead of the computa-
tional equation, 20 – 12 = 8. (See Van De Walle 2001, p. 111, for a discussion of semantic
and computational equations.) John was never dis-
couraged. He was motivated by his active involve-
ment and curiosity. Figure 1 shows his work.

The teacher gave the same problem to Mark. The
teacher and the student read the problem together.
Initially, Mark struggled with the problem; then he
slowly wrote the number sentence “20 – 8 = 12.” He started to work with the counters but abandoned
them and resorted to the number line. He placed one
finger on 20 and the other on 12; then he counted
the numbers between 12 and 20 and obtained 8, which he wrote in the blank space. The number sen-
tence he wrote was “20 – 8 = 12.” He seemed satis-
ified with his solution and very proud of his method.
He explained his method step by step to the teacher.
Figure 2 shows his solution.

To introduce some variety, the teacher gave Joan
a slightly different problem:

There were seventeen black cats in a tree. Some
of the cats jumped down. Now there are eight
cats in the tree. How many cats jumped down?

Joan carefully drew seventeen cats and crossed out
eight of them. The way she wrote her number sen-
tence, “17 – 9 = 8,” was consistent with the written
problem; that is, she wrote the semantic equation. When asked, she clearly explained her procedure
to the teacher. She said, “I started with seventeen
cats and some went away. There are eight now, so
I will mark them out and find out what went away.”
She carefully wrote her explanation on the problem
sheet. Figure 3 shows her solution.

Conclusion
This investigation has shown that children can be
very creative when they are allowed to choose
their methods for solving mathematical problems.
These first graders of different ability levels
solved problems that many people would consider
too advanced for them. They used different strate-
gies (modeling and counting) as well as different
representations (concrete, pictorial, and sym-

conic). The different strategies they used could be an indication of their levels of thinking. Mark
seemed to be the most advanced student because
he counted on the number line; the number line is more abstract than counters. Joan’s level of thinking seemed to be between those of Mark and John, and her explanation was the most fluent. Perhaps her explanation was indicative of her confidence and oral skills. John struggled a little before he obtained the solution. In his quest for the semantic equation, he appeared to use a trial-and-error approach, which he tried to reconcile with his previous knowledge.

Based on the teacher’s observations, we concluded that the students benefited immensely from the experiences. The students demonstrated a willingness to pursue the solutions and to share them with the teacher and the rest of the class, because it allowed them to be the teacher for a while. They learned from one another when they shared their solutions; they were able to see multiple solution strategies for the same problem. Some students learned to correct their own mistakes after seeing other solution strategies. John corrected what he thought was a mistake.

The students were actively engaged in the tasks; they were motivated and interested; they developed conceptual understanding, judging from their explanations; and they demonstrated improved social (collaboration) and communication (writing, reading, listening, and speaking) skills. They articulated their thinking clearly enough for the teacher to understand, and they did not appear inhibited in doing so. Their confidence was boosted; Mark, for example, appeared satisfied with and proud of his solution. We believe that the students’ reasoning and problem-solving skills improved; however, time and more of such problem-solving experiences may be necessary to see explicit evidence of the improvement of these skills.

The teacher’s reflections indicated that she, too, benefited from the experience. Most important, her approach to teaching young children and her interactions with them underwent significant changes. She has recognized the need to listen carefully to what students are saying, and she plans more thoroughly. She is using her knowledge of students’ thinking to plan appropriate tasks and questions. She has become more reflective about her practice with a view toward improving it. She is more tolerant of and open-minded to students’ solution strategies.

Occasionally, the teacher might have attempted to impose her thinking on the students. For example, in the case of John, the teacher might have insisted on the semantic equation. Although this is inconsistent with the philosophy of CGI, it is understandable in someone who is transitioning from a traditional teaching approach to a CGI approach. The teacher was at Level 2, Transitional, according to the levels in becoming a CGI teacher identified by Carpenter et al. (1999). At this level, she would use traditional and CGI approaches; so she would allow students to use their own strategies and also inject her strategies.

The results of this investigation lend support for a problem-solving approach to the teaching of mathematics. They are consistent with the findings of Riordan and Noyce (2001) and Reys et al. (2003), who concluded that children who are taught through a standards-based (usually problem-based) curriculum do better on standardized tests than do children who are taught through a traditional curriculum. The results also are consistent with the findings of the Treisman study cited by Singham (1998). Treisman found that a problem-solving approach to the teaching of mathematics resulted in improved grades for college students. A strong case for the use of a problem-solving approach in teaching mathematics is rapidly developing.
References


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