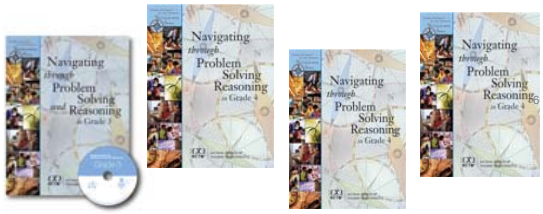
Navigating through Problem Solving and Reasoning

Building Future Problem Solvers

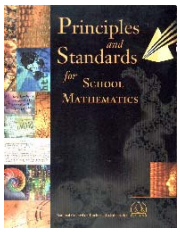
Seven grade level books (PreK-6th grades) that present investigations designed to develop students' reasoning and problem solving strategies.



by Michael Battista, Sally Mayberry, Denisse R. Thompson, Karol L. Yeatts, and Judith S. Zawajewski

NCTM Principles and Standards

- Five investigations are included in each grade level book.
- Each investigation is situated in a different one of the five content strands.

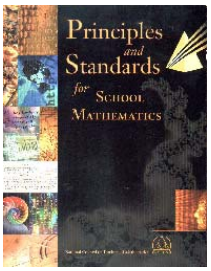


Content Standards

- Number and Operations
- Algebra
- Geometry
- Measurement
- Data Analysis and Probability

NCTM Process Standards

- Each investigation emphasizes the NCTM Process Standards.



Process Standards

- Problem Solving
- Communication
- Connections
- Representation
- Reasoning and Proof

Problem Solving

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Build new mathematical knowledge through problem solving
- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems
- Monitor and reflect on the process of mathematical problem solving

Reasoning and Proof

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

Communication

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Organize and consolidate their mathematical thinking through communication
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from prekindergarten through grade 12 should enable all students to—

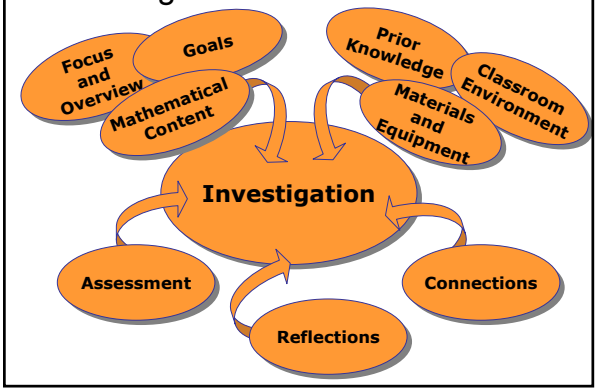
- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Representation

Instructional programs from prekindergarten through grade 12 should enable all students to—

- Create and use representations to organize, record, and communicate mathematical ideas
- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social, and mathematical phenomena

Organization of Lessons



Investigations

- Judith Fravillig (2001) has identified various teacher behaviors that are critical to helping students think deeply about mathematical ideas and share their thinking with others.
- These behaviors are summarized in three broad categories: eliciting students' thinking, supporting students' thinking, and extending students' thinking.
- **Fravillig, Judith. "Strategies for Advancing Children's Mathematical Thinking." *Teaching Children Mathematics*, April 2001 pages 454-459**

Eliciting students' thinking

- Elicit many solution methods for one problem.
- Wait for, and listen to, students' descriptions of solution methods.
- Encourage elaboration and student discussion.
- Use students' explanations as a basis for the lesson's content.
- Convey an attitude of acceptance toward students' errors and efforts.
- Promote collaborative problem solving.

Supporting **Extending**

Supporting students' thinking

Eliciting

- Remind students of conceptually similar problem situations.
- Provide background knowledge.
- Lead students through instant replays. (Revisit student solutions.)
- Write symbolic representations of solutions when appropriate.

Extending

Extending students' thinking

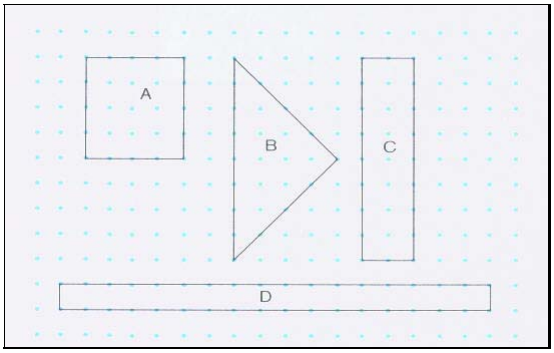
Eliciting

Supporting

- Maintain high standards and expectations for all students.
- Encourage students to draw generalizations.
- List all solution methods on the board to promote reflection.
- Push individual students to try alternative solution methods.
- Promote use of more-efficient solution methods.

Pose questions and tasks that elicit, engage, and challenge each student's thinking.

Which Pizza is Bigger?



Grade 3: Cut it Apart, Put it Together

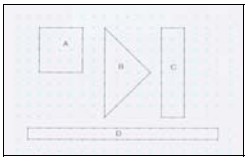
Ask students to clarify and justify their ideas orally and in writing.

Which Pizza is Bigger?

- Make a prediction for each problem.
- After you have made each prediction, check it by using scissors or with the 2-D Shape Decomposition Tool. (CD-ROM)
- For each pair of pizzas, predict which one is larger or if they are the same.
- Explain each answer.

Ask students to clarify and justify their ideas orally and in writing.

1. Pizza A or Pizza B
2. Pizza A or Pizza C
3. Pizza A or Pizza D
4. Pizza B or Pizza C
5. Pizza C or Pizza D
6. Pizza B or Pizza D



Ask a question, listen, ask another question

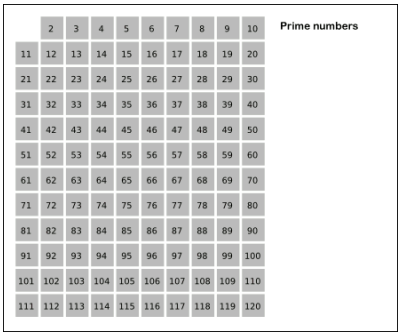
How many prime numbers exist between 1 and 100?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Grade 4: Discovering Primes As the Ancient Mathematicians Did

Ask a question, listen, ask another question

- What pattern did you notice when you crossed off the multiples of 2? The multiples of 3? The multiples of 4?
- Look at the pattern of crossed off numbers now. Does the pattern help you discover anything about the numbers that are multiples of 5?
- What other discoveries have you made as you crossed off more numbers?
- How many prime numbers did you find?



- http://nlvm.usu.edu/en/nav/frames_asid_158_g_2_t_1.html?open=instructions
- http://en.wikipedia.org/wiki/Sieve_of_Eratosthenes

Ask a question, listen, ask another question

- What is the first multiple of 7 that you found that you had not already crossed off as a multiple of a smaller prime?
- If you had just circle 11 as a prime and were crossing off multiples of 11, what number would you need to have in you chart to be able to cross off a multiple of 11 for the first time?
- What do you think are the next five prime numbers after 100?
- If you wanted to search for the primes in the numbers from 101 to 200, then from 201 to 300, how could you do this?

Decide what to pursue in depth from among the ideas that students bring up during a discussion.

Discovering Primes

- Students may still confuse the difference between prime and composite numbers.
- Allow them to explore with blocks, or paper squares.
- A prime number of blocks will form a rectangle in only one way.
  - one by the number of blocks
- A composite number of blocks will form a rectangle in more than one way
  - 12 blocks will form a rectangle that is
    - 1 x 12 2 x 6 3 x 4

Grade 4: Discovering Primes As the Ancient Mathematicians Did

Listen carefully to students' reasoning

What is Jamie's Secret Pin Number?

- Jamie has recently installed a new alarm system and needs to create a secret password (PIN number) for the alarm.
- She decides to use the numbers in her birthday: **05-24-76** for her six digit PIN number.
- It is of the utmost importance to Jamie that no one else be able to decipher her secret code so she decides to rearrange this special 6 digit number in a way that only she can remember it.

Grade 5: Pin Numbers and Secret Codes

Listen carefully to students' reasoning

After much thought, Jamie decides to rearrange the numbers **05-24-76** so that:

- The first digit is divisible by 1;
- Its first two digits are divisible by 2;
- Its first three digits are divisible by 3;
- Its first four digits are divisible by 4;
- Its first five digits are divisible by 5; and
- The entire 6-digit number is divisible by 6.

What is Jamie's 6-digit PIN number?

Listen carefully to students' reasoning

702456 is a solution because:

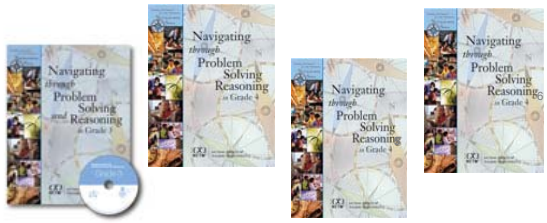
- 1st digit 7 is divisible by 1.
- 1st and 2nd digits together (70) are divisible by 2 because 70 is an even number.
- 1st, 2nd and 3rd digits together (702) are divisible by 3 because the sum of the digits 7+0+2=9 and 9 is divisible by 3.
- 1st, 2nd, 3rd and 4th digits together (7024) are divisible by 4 because the last two digits (24) are divisible by 4.
- 1st, 2nd, 3rd, 4th and 5th digits together (70245) are divisible by 5 because the last digit ends in a 5.
- 1st, 2nd, 3rd, 4th, 5th, and 6th digits together (702456) are divisible by 6 because it is also divisible by 2 and 3. It is divisible by 2 because it ends in an even number. It is divisible by 3 because the sum of the digits (7+0+2+4+5+6=24) is divisible by 3.
- NOTE: The secret PIN number can also be 720456 or 726450.

Practicing Effective Questioning

Maximize...	Minimize...
...asking questions that begin with words like "What if," "Explain," "Analyze," "Create," and "Compare and contrast," etc.	...asking questions that have a "yes" or "no" response and questions that require merely direct recall of definitions etc.
...the amount of time you wait after you pose a question, i.e. wait-time, in order to allow students to process the question in their minds.	...calling on students directly after you pose a question and calling on a student before you even ask the question.
...asking students to elaborate on their answers and asking students "why."	...telling a student their answer is wrong and not asking them to think of why it is wrong.
...opportunities for students to pose questions among themselves.	...straight lecture without student interaction.
...providing opportunities that challenge students' original conceptual understandings.	...providing opportunities that do not encourage creative and critical thinking.
...encouraging students to work through their decision making process, even if it bring frustration and makes them leave their comfort zone of learning.	...giving students direct answers to their questions without allowing them to think through the decision making process.

http://www.ndt-ed.org/TeachingResources/ClassroomTips/Effective_Questioning.htm

Navigating through Problem Solving and Reasoning are available at the NCTM Conference Book Store



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