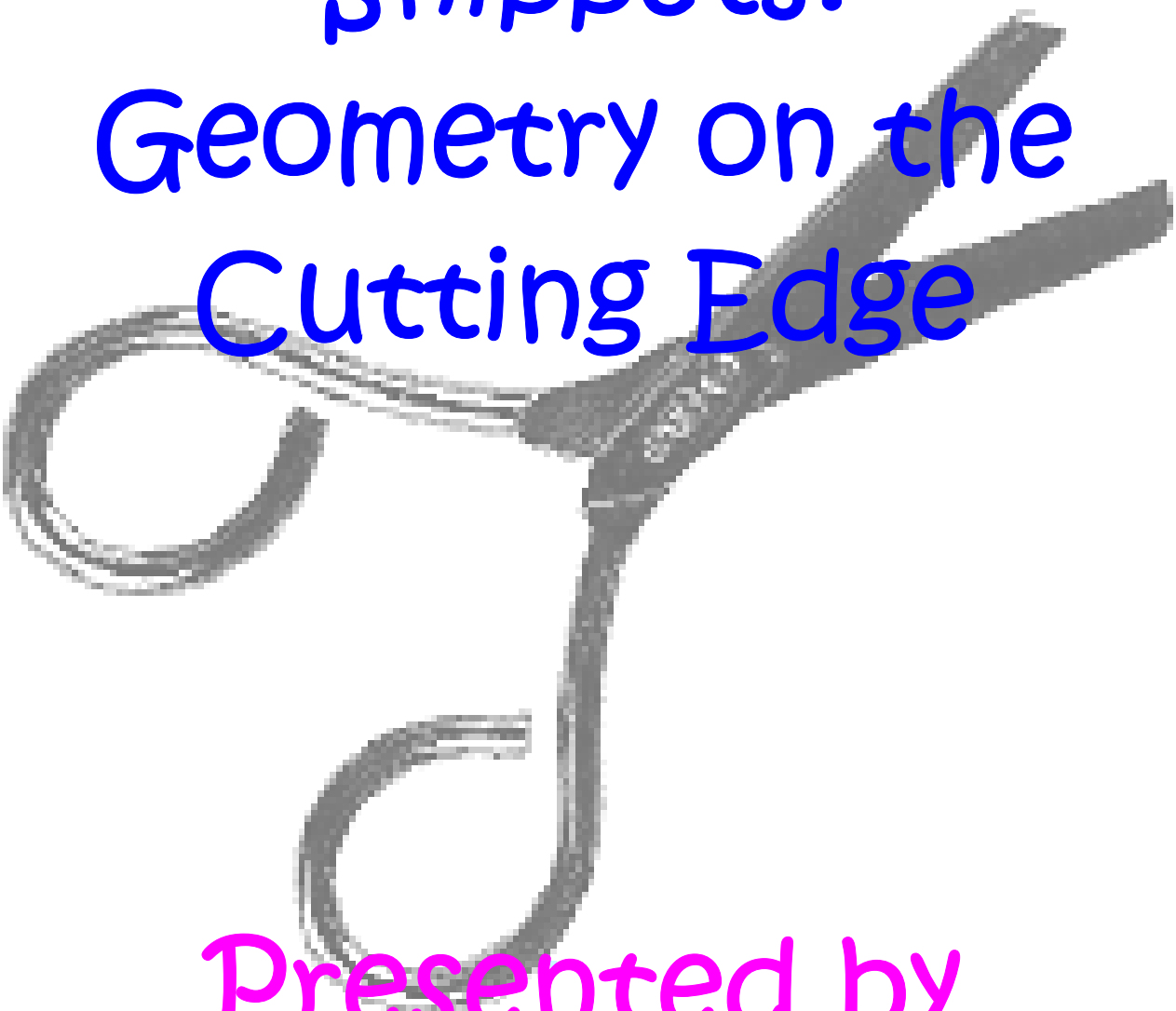
Snippets: Geometry on the Cutting Edge



Presented by

Dr. Karol Yeatts

NCTM Geometry Standard

In prekindergarten through grade 2 all students should—

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- recognize, name, build, draw, compare, and sort two- and three-dimensional shapes;
- describe attributes and parts of two- and three-dimensional shapes;
- investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.

Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- describe, name, and interpret relative positions in space and apply ideas about relative position;
- describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance;
- find and name locations with simple relationships such as "near to" and in coordinate systems such as maps.

Apply transformations and use symmetry to analyze mathematical situations
- recognize and apply slides, flips, and turns;
- recognize and create shapes that have symmetry.

Use visualization, spatial reasoning, and geometric modeling to solve problems
- create mental images of geometric shapes using spatial memory and spatial visualization;
- recognize and represent shapes from different perspectives; relate ideas in geometry to ideas in number and measurement;
- recognize geometric shapes and structures in the environment and specify their location.

In grades 3-5 all students should—

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes;
- classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids;
- investigate, describe, and reason about the results of subdividing, combining, and transforming shapes;
- explore congruence and similarity;
- make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.

Specify locations and describe spatial relationships using coordinate geometry and other representational systems
- describe location and movement using common language and geometric vocabulary;
- make and use coordinate systems to specify locations and to describe paths;
- find the distance between points along horizontal and vertical lines of a coordinate system.

Apply transformations and use symmetry to analyze mathematical situations
- predict and describe the results of sliding, flipping, and turning two-dimensional shapes;
- describe a motion or a series of motions that will show that two shapes are congruent;
- identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs.

Use visualization, spatial reasoning, and geometric modeling to solve problems
- build and draw geometric objects;
- create and describe mental images of objects, patterns, and paths;
- identify and build a three-dimensional object from two-dimensional representations of that object; identify and draw a two-dimensional representation of a three-dimensional object;
- use geometric models to solve problems in other areas of mathematics, such as number and measurement;
- recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life.

Shapes from Shapes

- Adapted from Navigating through Geometry in Prekindergarten-Grade 2 pages 14-16.

NCTM Standard and Expectation

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
  - investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.

Materials
Paper Triangles, Several sheets of drawing paper, pencils, crayons

Engage

- Have pre-cut triangles available for children to use.
- Begin by demonstrating on the overhead projector how to put 4 triangles together to make a larger triangle.
- Ask students to describe what the larger shape looks like.
- Demonstrate how the 4 triangles can be moved around to make a different shape.
- Have students tell what the different designs look like to them.
- After demonstrating several examples, have children work in groups to create different designs.
- Have students name the group's design.
- Have students draw their shape designs
- Provide an opportunity for groups to share their designs with the class.
- To extend the activity, have students create different designs using different shapes.

Constructing Polygons

NCTM Standard and Expectation

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
  - Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes;
  - investigate, describe, and reason about the results of subdividing, combining, and transforming shapes.

Materials
2" paper squares, Scissors, Chart paper, tape

Engage

- Have students work in groups.
- Give students lots of 2" paper squares.
- Have students fold the paper square along a diagonal to divide the square into two triangular regions.
- Next, fold the triangle in half again to divide the square into four triangular regions.
- Have students cut along each fold line to make four congruent triangles.
- Have students arrange the triangles to create as many polygons as possible. NOTE: The sides of each triangle must touch and may not overlap.
- Post the different polygons on a large class chart.
- Identify the polygons created by identifying the number of angles and sides of each polygon.

Cutting Corners

- Adapted from Navigating through Geometry in Prekindergarten-Grade 2 pages 22-26.

NCTM Standard and Expectation

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
  - describe the attributes and parts of two- and three-dimensional shapes;
  - investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.

Materials
Large chart (the blackboard or bulletin board may be used), Scissors, 8 ½" x 11" white paper, glue/tape construction paper, rulers, pencils

Engage

- Construct a large chart that has four columns. Label the columns: Shape name, Number of straight sides, number of vertices, congruent.
- Hold up a rectangular piece of paper and ask students to identify the shape.
- Ask how many sides does the rectangle have. How many vertices does the rectangle have.
- Ask how you can cut the rectangle to get two smaller rectangles whose shape and size are the same.
- Demonstrate how to compare the rectangles to show that they are the same shape and size. Introduce the term congruent.
- Hold up a new piece of paper. Ask if there is another way to cut this rectangle to get two congruent rectangles.
- Using a new sheet of paper, fold and cut along a diagonal. Have students identify the shapes and whether they are congruent.
- Using a new sheet, tell students that you are going to make one straight cut from one side to the other side. Point to the starting and ending positions. Have student predict what the new shapes will be and whether they will be congruent.
- Next, cut off a corner of the rectangular piece of paper leaving a small triangle and a large pentagon.
- Have students identify the number of sides and vertices of each shape.
- Place the new shapes on a large chart and record the name of the shape, the number of sides and vertices of the shapes in the appropriate columns.
- Have students cut pieces of paper to create shapes by drawing a straight line from one side or vertex to the other side or vertex. Note: Students may use a ruler or a straight edge tool to draw straight lines.
- Have student glue/tape their new shapes on the chart and record how many sides and vertices each shape has. Help children identify the name of the shape.
- Extend the activity by having students use an equilateral triangle or a trapezoid and follow the same procedures naming the shapes, identifying the number of sides and vertices of each newly cut shape and whether the shapes are congruent.

Folding Shapes

- Adapted from Navigating through Geometry in Prekindergarten-Grade 2 pages 59-61.

NCTM Standard and Expectation

- Apply transformations and use symmetry to analyze mathematical situations.
  - Recognize and create shapes that have symmetry.

Materials

Paper cut-outs of large circles, squares, nonsquare rectangles, equilateral triangles, isosceles triangles, regular pentagons and hexagons, crayons, mirrors, large chart (chalkboard)

Engage

- Prior to beginning the activity, create a chart with two columns. Label the first column, Shape Name and the second column, Number of Lines of Symmetry.
- Give each student a square. Have students fold the square in half.
- Open the paper and point to the fold line. Explain that this is called the line of symmetry.
- Have students use a crayon to draw their fold line.
- Explain that the square has more than one line of symmetry.
- Encourage students to find the other lines of symmetry on their square.
- Have students use a different color crayon to draw the other lines of symmetry. Note that a square has four lines of symmetry.
- Give students an equilateral triangle, a regular pentagon and regular hexagon.
- Say the names of each shape.
- Have students identify and draw the lines of symmetry for each shape.
- Record the number of lines on symmetry for each shape on a large chart.
- Have students discuss the results displayed on the chart that show the number of lines of symmetry for the shapes.
- Students may observe that the number of lines of symmetry is equal to the number of sides of the regular shapes they have examined.
- To extend the activity, provide students with a nonsquare rectangle and a nonequilalteral isosceles triangle.
- Have students identify the number of lines of symmetry.
- Discuss the results compared to the previous results.
- Some students may observe that these shapes do not have all the same size lines so the number of sides and the number of lines of symmetry are not equal.
- Provide students with a circle and have students identify the number of lines of symmetry.

Symmetry of Pentominoes

- Adapted from Navigating through Geometry in Grades 3-5 pages 59-63.

NCTM Standard and Expectation

- Apply transformations and use symmetry to analyze mathematical situations.
  - Predict and describe the results of sliding, flipping, and turning two-dimensional shapes
  - Identify and describe line and rotational symmetry in two- and three- dimensional shapes and designs

Materials

1" squares (color tiles), 1" grid paper, Tape, Scissors, Pencils, Straight edge/ruler

Engage

- Explain to students that a pentomino is a shape formed by joining 5 squares of equal size.
- Have students work with a partner and use the 1" paper squares to discover how many different formations of pentominoes are possible.
- Have student construct the various pentomino configurations by taping their paper squares together. You may also have students shade in the 1" grid paper to represent the various pentominoes.
- Demonstrate on the overhead projector the rules for arranging the squares. Each square must share a side of another square completely.

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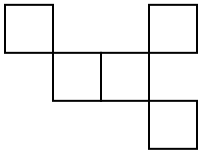
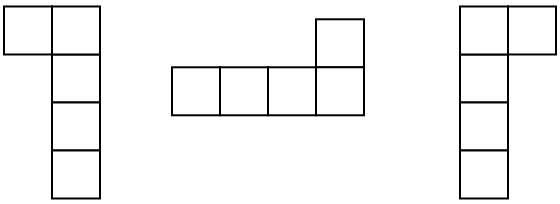
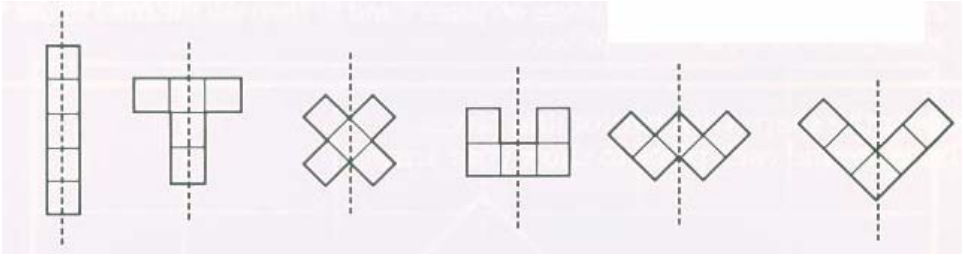
NOT ALLOWED 

- A shape that has been rotated or reflected still counts as only 1 pentomino.

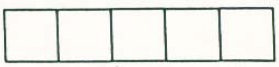


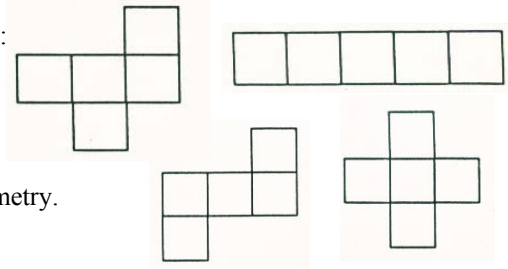
Explore

- With the pentominoes, have students determine which pentominoes have line symmetry.
- Have students trace each pentomino onto the 1" grid paper and then cut out each pentomino.
- Have students predict which shapes have line symmetry.
- To test for line symmetry, have students fold each of the cutout pentominoes in half vertically, horizontally and on the diagonal and compare the two parts.
- Have students use a ruler to draw the line(s) of symmetry on each pentomino.
- Have students discuss their findings.



- Explain what rotational symmetry means. "If a figure can be rotated about a point in such a way that its rotated image coincides with the original figure after turning less than 360 degrees, then the figure has rotational symmetry." Michael Serra, Discovering Geometry by Key Curriculum.
- Using a sheet of acetate, make two outlines of the following pentomino pieces:
- Place one acetate on top of the other and then rotate the top piece a half turn.
- Ask which piece fits back in its outline.





- Explain that this piece has rotational symmetry.
- Have students determine which of the other pentominoes have rotational symmetry.
- Have students discuss their findings.

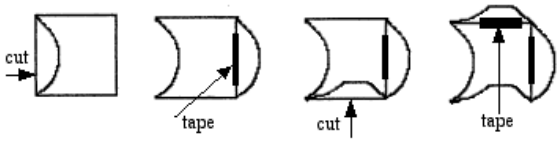
Zany Tessellations

- Adapted from Navigating through Geometry in Grades 3-5 pages 68-74.

NCTM Standard and Expectation

- Apply transformations and use symmetry to analyze mathematical situations.
  - Predict and describe the results of sliding, flipping, and turning two-dimensional shapes

Materials
Index cards, Construction paper, Tape, Scissors, Books of M.C. Escher art or samples of Escher's work
http://www.thinkquest.org/library/index.html -go to Math then Geometry then click on **Totally Tessellated: An Introduction to Tessellations**

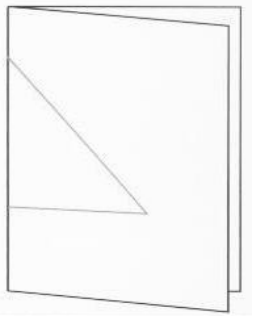


Engage

- Cut a small square (2"x 2" or 3"x 3") from index cards.
- Draw a line beginning at the top left corner of the small square and ending at the bottom left corner.
- Carefully cut along this line. Then, attach the straight edge of the cut out piece to the opposite side of the small square.
- Next, make another line beginning at the bottom left corner of the small square and ending at the bottom right corner.
- Again, cut along the line. Then, attach the straight edge of the cut out piece to the top straight edge of the small square.
- Trace the new figure on a piece of paper and cut it out.
- Continue tracing and cutting out additional pieces until there are ten or more identical pieces.
- Arrange the pieces on a large sheet of paper such that the pieces fit together and form a tessellating pattern.
- Glue the pieces on the large piece on paper and add any finishing touches.

Cutouts

Adapted from Navigating through Geometry in Prekindergarten-Grade 2 pages 73-75



NCTM Standard and Expectation

- Use visualization, spatial reasoning, and geometric modeling to solve problems.
  - Create mental images of geometric shapes using spatial memory and spatial visualization.

Materials
Paper, scissors, folded paper with pre-drawn shapes

Engage

- Fold a piece of paper in half and draw a right triangle along the fold line.
- Ask students to predict what the shape will look like when you cut it on along the lines and unfold the cutout.
- Unfold the cutout and talk about how the original shape is like the cut-and unfolded shape and how the two shapes differ.
- Give students a sheet of folded paper with a designed drawn along the fold line.
- Have students draw a picture of what they think the shapes will look like when they are cut out and unfolded.
- Have students cut out their shapes to test their predictions.
- Have students make their own simple design on the fold line and predict what it will be when they cut it out.

Imagine Maker

Adapted from: Wheatley, Grayson H. and Reynolds, Anne M. "Image Maker" Developing Spatial Sense. Teaching Children Mathematics: Reston, VA: NCTM, (February, 1999) 374-378.

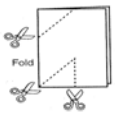
NCTM Standard and Expectation

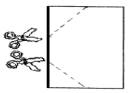
- Use visualization, spatial reasoning, and geometric modeling to solve problems.
  - Create and describe mental images of objects, patterns, and paths.

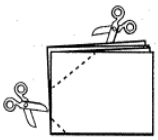
Engage

- Begin by folding one of the 8" x 8" paper squares in half.
- Draw a shape on one side of the folded paper. It might be half a heart or tree.
- Ask students to imagine what the paper will look like if you cut along the outline and then unfold the paper.
- Have students describe what the paper will look like when unfolded.
- Cut along the outline shape.
- Unfold the paper and show how it looks.

- Distribute unlined paper to the students.
- Ask students to fold the paper into four equivalent regions.
- Next fold one of the 8" x 8" paper squares in half.
- Have students notice that the fold line is vertical.
- Cut the paper as shown along the dotted lines.
- Ask students to draw what this paper will look like when it is unfolded.
- Unfold the paper and ask students to compare their drawing with the unfolded paper.
- Ask students, "How did you draw the shape?"
- Repeat the procedure of folding and cutting the paper as shown for the next two problems.
- Next, fold the paper to create a double fold.
- Cut along the dotted lines and have students draw what the paper will look like when it is unfolded.
- Again, ask students to describe how they drew the shape.





Tangram Challenges

- Adapted from Navigating through Geometry in Grades 3-5 pages 77-79.
- *Grandfather Tang's Story* by Ann Tompert. Crown Publishers, Inc. New York 1990.

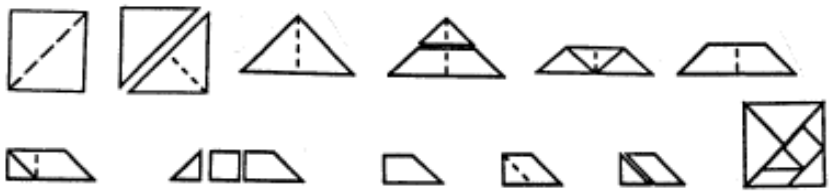
NCTM Standard and Expectation

- Use visualization, spatial reasoning, and geometric modeling to solve problems.
  - Create and describe mental images of objects, patterns, and paths.

Materials
6" x 6" paper squares, scissors

Engage

- Read *Grandfather Tang's Story* by Ann Tompert.
- Give each student a 6" x 6" paper square.
- Have students fold the square along its diagonal and cut along the fold.
- Take one of the triangles and fold it in half. Then unfold it and cut along the fold.
- Set the two smaller triangles aside. Take the large triangle and fold it in half. Open it up and fold the top half down until its tip meets the bottom center.
- Cut along the second fold. Set aside the small triangle piece that was cut from the top of the larger triangle.
- The piece that is left is a trapezoid. Hold it with its longest side toward you. It still has a fold line down the middle.
- Fold the lower left edge corner so that the corner meets at the bottom of the center fold line. Unfold it and cut along the diagonal fold and the center fold lines. The shapes cut are a square and a small triangle. The remaining shape is a trapezoid.
- Take the trapezoid and fold it to form a triangle and a parallelogram by folding the bottom left corner up to meet the top right corner. Cut along the fold to create the sixth and seventh tangram pieces.
- Have students create their own tangram creatures using their tangram pieces.
- Challenge students use their tangram pieces to create various polygons. For example:
  Create a square using 1, 2, 3, 4, 5, 6, or 7 pieces. Create a triangle using 1, 2, 3, 4, 5, 6, or 7 pieces.
  Create a non square rectangle using 1, 2, 3, 4, 5, 6, or 7 pieces. Create a pentagon using 1, 2, 3, 4, 5, 6, or 7 pieces



Picture Pie Activity

- Emberley, Ed (1984). Picture Pie. Boston: MA. Little, Brown and Company.
- Emberley, Ed (1996). Picture Pie 2. Boston: MA. Little, Brown and Company.

NCTM Standard and Expectation

- Use visualization, spatial reasoning, and geometric modeling to solve problems.
  - Use geometric models to solve problems in other areas of mathematics, such as number and measurement.

Materials
A large supple of construction paper, Scissors, Rulers, Pencils, Compass, Overhead projector and markers

Engage

- Read Picture Pie by Ed Emberley.
- Give students construction paper.
- Have students use a compass to draw a circle on one of the pieces of construction paper.
- Have students carefully cut out the circle. Have students fold the circle in half and identify each section as half of the circle. Cut the circle in half along the fold line.
- Have students fold one of the half circles in half. Identify the new section as one fourth. Cut the half circle along the fold line to create two fourths.
- Have students fold one of the fourths in half. Identify the new sections formed as eighths. Have students place all the pieces together to make a whole circle.
- Ask students to describe the relationships between the sections.
  How many half circles are needed to make a whole circle? How many fourths are needed to make half of a circle?
  How many fourths are needed to make a whole circle? How many eighths are needed to make half of a circle?
  How many eighths are needed to make a whole circle? How many eighths are needed to make one fourth of a circle?
- Have students create a picture using halves, fourths, and eighths of circles.
- Have students identify the fractional parts used to create their designs.

Fraction Fantasy

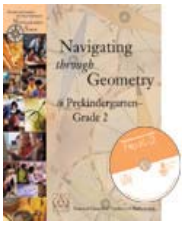
- Adapted from Navigating through Geometry in Grades 3-5 pages 88-89.

NCTM Standard and Expectation
- Use visualization, spatial reasoning, and geometric modeling to solve problems.
  - Use geometric models to solve problems in other areas of mathematics, such as number and measurement.

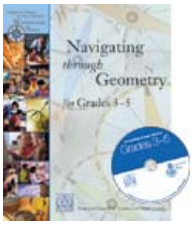
Materials
A large supply of paper squares, scissors, rulers, pencils

Engage
- Give students one paper square.
- Ask students to cut a model for the fraction one-half from the square to form two congruent pieces.
- Give the students another square and ask them to create a different model for one-half that is not congruent to the models they just created.
- Have students continue to cut out as many additional representations for one-half as they can that are not congruent to any they have previously created.
- Once students have created several models for halves, have them move on to presenting thirds, fourths, sixths, eight, tenths, and twelfths.
- Ask students to tell how they can prove that two fractional pieces are equal parts of the same whole? (If the parts are congruent, use transformations to show that if rotated, reflected or translated, the parts can be made to coincide.)

NCTM Navigating Through Geometry



By Carol R. Findell, Marian Small, Mary Cavanagh, Linda Dacey,Carole E. Greenes, and Linda Jensen Sheffield
Focusing on the important ideas of geometry, this book shows how to investigate two- and three-dimensional shapes with very young children. It introduces methods to describe location and position, explores simple transformations, and addresses visualization, spatial reasoning, and the building and drawing of constructions. Activities in each chapter pose questions that stimulate students to think more deeply about mathematical ideas. The CD-ROM features fourteen articles from NCTM publications. The supplemental CD-ROM also features interactive electronic activities, master copies of handouts, and additional readings.



By M. Katherine Gavin, Louise P. Belkin, Ann Marie Spinelli, and Judy St. Marie
The "big ideas" of geometry–shape, location, transformations, and spatial visualization–are the focus of this book. Sequential activities will enrich the curriculum and help students develop a strong sense of geometric concepts and relationships, leading them to "grasp the space in which they live." The supplemental CD-ROM features interactive electronic activities, master copies of handouts, and additional readings.